COMPRESSION OF MAGNETIC FIELD IN AN IMPLODING SPHERICAL CAVITY

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The equations describing the compression of a magnetic field, produced by a system of meridional currents, in an imploding spherical cavity are solved in the present communication.

The compression of a magnetic field between two approaching perfectly conducting surfaces is investigated by the method of characteristics in [1, 2]. The case of a cylindrical geometry is examined in [2] by means of integral transformations. The solutions of these problems have a bearing on the physical processes occurring in systems in which a certain "favored" dimension can be ascertained.

In a number of cases (in the experiments described in [3], for example) the most important, final stage of compression occurs in a volume which has no particularly well expressed favored dimension. In this connection we consider below the compression of a magnetic field in a spherical cavity. Unlike the corresponding problem for a cylindrical geometry [2], the present problem permits of a simple and readily comprehensible solution.

1. The electrodynamic problem to be considered is that of the compression of an initially produced magnetic field in an imploding spherical cavity in a perfectly conducting medium. We restrict the discussion to the case when the radius of the cavity varies linearly with time:

$$a(t) = a_0 - vt \tag{1.1}$$

where a_0 is the initial radius of the cavity.

The experimental variation of a(t) reported in [1] indicates that (1.1) is obeyed quite well in practice right up to a time approaching the moment of maximum compression.

As the initial field we take the field produced by an axially symmetric system of currents (shown in Fig. 1). The surface meridional currents j (total current I) enter at A a wire AB across the diameter of the cavity and leave the wire at B.

In a spherical coordinate system with origin at the center of the sphere it follows from symmetry considerations that the only nonzero components of the electromagnetic field are H_φ, E_ϑ, and E_r. It follows immediately from Maxwell's equations that time derivatives of all orders of the radial component are zero, so that this component is not excited during implosion.

We thus arrive at the following problem. It is required to solve Maxwell's equations subject to the initial conditions

$$H_\varphi(r, \vartheta, 0) = \begin{cases} Ir^{-1}(\sin\vartheta)^{-1} & (r \leqslant a_0) \\ 0 & (r > a_0) \end{cases} \qquad E_\vartheta(r, \vartheta, 0) = 0 \tag{1.2}$$

and the boundary condition

$$E_\vartheta(a(t), \vartheta, t) - \dot{a}c^{-1} H_\varphi(a(t), \vartheta, t) = 0 \tag{1.3}$$

The latter condition must hold at a perfectly conducting surface [4] moving with a velocity da/dt.

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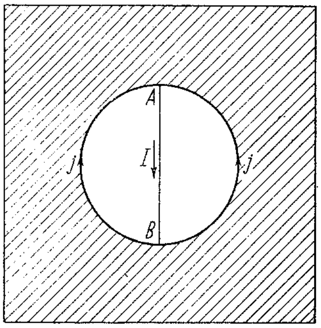

Fig. 1

Maxwell's equations, in conjunction with the boundary and the initial conditions, permit separation of the angular variable if we put

$$H_\varphi(r, \vartheta, t) = (\sin\vartheta)^{-1} h(r, t), \quad E_\vartheta(r, \vartheta, t) = (\sin\vartheta)^{-1} e(r, t) \tag{1.4}$$

2. Separating out the angular dependence and eliminating the function e (r, t) from the equations and the boundary and initial conditions, we arrive at the problem

$$\frac{\partial^2 h}{\partial r^2} + \frac{2}{r}\frac{\partial h}{\partial r} - \frac{1}{c^2}\frac{\partial^2 h}{\partial t^2} = 0, \quad h(r, 0) = \frac{I}{r}, \quad \left(\frac{\partial h}{\partial t}\right)_{t=0} = 0$$

$$\left(\frac{\partial h}{\partial r}\right)_{r=a} + \frac{a'}{c^2}\left(\frac{\partial h}{\partial t}\right)_{r=a} + \frac{1}{c^2}\frac{d}{dt}[a' h(a, t)] = -(1 + \beta^2)\frac{h(a, t)}{a} \tag{2.1}$$

$$(\beta = v / c = -a' / c)$$

We solve (2.1) with the aid of a Fourier-Bessel transformation:

$$\Phi(k, t) = \int_0^{a(t)} h(r, t)\, n_0(kr)\, r^2 dr \qquad \left(n_0(kr) = -\frac{\cos kr}{kr}\right)$$

$$h(r, t) = 2\pi^{-1}\int_0^\infty \Phi(k, t)\, n_0(kr)\, k^2 dk \tag{2.2}$$

Here n_0 is the spherical Neumann function of order zero. The choice of $n_0(kr)$ as eigenfunction is dictated by the singularity in the initial distribution (1.2) at r = 0.

Application of transformation (2.2) reduces (2.1) to an equation for $\Phi(k, t)$

$$d^2\Phi / dt^2 + k^2c^2\Phi = -c^2(1 - \beta^2)\, ka^2\, j_0'(ka)\, h(a, t) \tag{2.3}$$

with the initial conditions

$$\Phi(k, 0) = -Ik^{-2}\sin ka_0, \quad (d\Phi / dt)_{t=0} = 0 \tag{2.4}$$

The solution satisfying (2.3) and (2.4) has the form

$$\Phi(k, t) = -Ik^{-2}\sin ka_0 \cos kct - c(1 - \beta^2)\int_0^t a^2(\tau)\, h^*(\tau)\, j_0'(ka(\tau))\sin kc(t - \tau)\, d\tau$$

$$(h^*(t) = h(a(t), t)) \tag{2.5}$$

The unknown field at the boundary of the cavity appears in (2.5). To determine this field we put r = a(t) in the inversion formula of (2.2) and double the integral on the right side since

$$h^*(t) = 2^{-1}[h(a(t) + 0, t) + h(a(t) - 0, t)], \quad h(a(t) + 0, t) = 0$$

This gives the following functional equation for $h^*(t)$:

$$h^*(t) = \frac{I}{(1 - \beta)\, a(t)} \qquad \left(0 < t < \frac{2\beta T}{1 + \beta} = t_1, \; T = \frac{a_0}{v}\right)$$

$$h^*(t) = \gamma^2 h^*(\gamma(t - t_1)) \qquad (t_1 < t < T, \; \gamma = (1 + \beta) / (1 - \beta)) \tag{2.6}$$

We define a time sequence $\{t_n\}$ in the following manner:

$$c(t_n - t_{n-1}) = a(t_n) + a(t_{n-1}) \tag{2.7}$$

From (2.7) we obtain

$$t_n = T(1 - \gamma^{-n}) \qquad (n = 0, 1, 2, \ldots)$$

The sequence $\{t_n\}$ converges to T as $n \to \infty$. The differences $t_n - t_{n-1}$ correspond to the times taken by the electromagnetic wave to traverse the path wall-center-wall. The solution of Eq. (2.6) thus has the form

$$h^*(t) = \gamma^{n-1}(1 - \beta)^{-1} I\, [a(t)]^{-1} \qquad (t_{n-1} < t < t_n) \tag{2.8}$$

The required solution h(r, t) is obtained by substituting (2.8) into (2.5) and the latter into (2.2):

$$h(r,t)=\begin{cases} Ir^{-1}, & 0<t<c^{-1}(a_0-r) \\ {}^1/_2 Ir^{-1}\gamma^n(\gamma+1), & t_n+c^{-1}[a(t_n)-r]<t<t_n+c^{-1}[a(t_n)+r] \\ Ir^{-1}\gamma^{n+1}, & t_n+c^{-1}[a(t_n)+r]<t<t_{n+1}+c^{-1}[a(t_{n+1})-r] \end{cases}$$

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